Ex. 3. Show that the set $\{1, \omega, \omega^2\}$, where ω is an imaginary cube root of unity forms a finite abelian group under multiplication.

(*Bundelkhand 79 ; Kanpur 84, 80 ; Lucknow 81, 79 ; Rohilkhand 82, 80*)

Solution. Cube roots of unity are obtained by solving the equation $x^3=1$ or $x^3-1=0$ or $(x-1)(x^2+x+1)=0$

or $$x=1,\ x=\frac{-1\pm\sqrt{(1-4)}}{2}=\frac{-1\pm i\sqrt{3}}{2}$$

Put $$-\tfrac{1}{2}+\frac{i\sqrt{3}}{2}=\omega$$

Then $$-\tfrac{1}{2}-\frac{i\sqrt{3}}{2}=\omega^2 \text{ and } \omega^3=1$$

Hence cube roots of unity are 1, ω and ω^2.

All possible multiplications in the form of the composition table are given below (using the fact $\omega^3=1$, $\omega^4=\omega^3.\omega=\omega$).

$\cdot$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

P_1 : This table shows that multiplication is a binary operation on the given set as the product of any two elements of the given set under multiplication belongs to the set *i.e.* closure property holds.

P_2 : From the above table we find that

$$(1.\omega).\omega^2=\omega.\omega^2=1 \text{ and } 1.(\omega.\omega^2)=1.\omega^3=1\ .\ 1=1$$

i.e. $(1.\omega).\omega^2=1.(\omega.\omega^2)$ *i.e.* the associative law holds.

P_3 : From the table it is evident that 1 is the identity.

P_4 : From the table we find that the inverses of 1, ω and ω^2 are 1, ω^2 and ω respectively, since

$$1\ .\ 1=1\ ;\ \omega.\omega^2=\omega^3=1 \text{ and } \omega^2.\omega=\omega^3=1.$$

P_5 : $x\ .\ y=y\ .\ x\ \forall\ x, y\in M$. where $M=\{1, \omega, \omega^2\}$.

Hence all the group postulates are satisfied and so M is a finite abelian group under multiplication.

***Ex. 4. Prove that the set {0, 1, 2, 3} is a finite abelian group of order 4 under addition modulo 4 as composition.**

Solution. The composition table is

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

P_1 : The above table shows that the sum of any two elements of the given set belongs to the set and therefore under the given operation the set is closed.

P_2 : $(a+b)+c$ and $a+(b+c)$ both denote zero or the least non-negative integer obtained on dividing the ordinary sum of a, b and c by 4 *i.e.* the composition is **associative.**

P_3 : From the table it is evident that 0 is the identity.

P_4 : The inverses of 0, 1, 2, 3, are 0, 3, 2, 1

since
$$0+0=0 \; ; \; 1+3=4\equiv 0 \text{ (mod. 4)}$$
$$2+2=4\equiv 0 \text{ (mod 4)} \; ; \; 3+1=4\equiv 0 \text{ (mod. 4)}$$
and which is also evident from the table. **(Note)**

P_5 : $a+b=b+a$ for any two elements a, b in the given set.

Hence all the group postulates are satisfied and so the given set is a finite abelian group of order 4 under the given composition.

Ex. 5 (a) Show that th